

Classical stability of stringy wormholes in flat and AdS spaces

Jin Young Kim^{*}

Department of Physics, Kunsan National University, Kunsan 573-701, Korea

Yoonbai Kim[†]

BK21 Physics Research Division and Institute of Basic Science,

Sungkyunkwan University, Suwon 440-746, Korea

School of Physics, Korea Institute for Advanced Study, 207-43,

Cheongryangri-Dong, Dongdaemun-Gu, Seoul 130-012, Korea

James E. Hetrick[‡]

Department of Physics, University of the Pacific,

3601 Pacific Avenue, Stockton, CA 95211

(Dated: February 3, 2003)

Abstract

We study small fluctuations of the stringy wormhole solutions of graviton-dilaton-axion system in arbitrary dimensions. We show under $O(d)$ -symmetric harmonic perturbation that the Euclidean wormhole solutions are unstable in flat space irrespective of dimensions and in anti de Sitter space of $d = 3$.

PACS numbers: 04.50.+h, 11.25.-w, 98.80.Cq

^{*} Electronic address: jykim@kunsan.ac.kr

[†] Electronic address: yoonbai@skku.ac.kr

[‡] Electronic address: jhetrick@uop.edu

I. INTRODUCTION

Wormholes can play role in many interesting phenomena in quantum gravity. It is known that there are various kinds of Euclidean wormhole solutions. In four dimensions the following matter fields were adopted to support the throat of the wormhole, e.g., axion field [1], scalar fields [2], SU(2) Yang-Mills fields [3]. Higher dimensional wormhole solutions were obtained [4], and a higher derivative correction to the Einstein-Hilbert action was considered [5]. Wormhole solutions based on string theory were studied in Ref. [6, 7] where massless dilaton, in addition to the axion, was naturally included. For stringy wormholes, the existence of non-singular solutions with finite action depends on how the dilaton couples to the axion. Recently Gutperle and Sabra constructed instanton and wormhole solutions in dilaton-axion-graviton sector of d -dimensional supergravity theory induced from string theory [8]. They found the critical value for coupling below which non-singular solutions exist.

Wormhole configurations make non-trivial contributions to the functional integral in quantum gravity. Wormholes are usually assumed to make real contribution to amplitudes in quantum gravity. However, Rubakov and Shvedov [9] found a negative mode among fluctuations about the Giddings-Strominger wormhole solution [1] in the context of semiclassical approach, which implies that the wormhole contribution to the Euclidean functional integral involves imaginary contribution. This suggests the interpretation of the wormhole as describing the instability of a large universe against the emission of baby universes. Similar analysis of the stringy wormholes were carried out [10] under the assumption of a linear relation between perturbed fields. They confirmed the existence of continuous spectrum of the negative modes about the nonsingular wormhole background.

In this paper, we will consider small fluctuations around the wormhole solutions in d -dimensions. To look into the signature of lowest modes, we use s -wave perturbation with $O(d)$ symmetry and obtain coupled linear equations. Similar to the stability analysis of black holes [11], harmonic form of perturbation is assumed and it provides the algebraic constraint for the stability, depending on couplings, the value of cosmological constant, spacetime dimensions, and integration constants. It is confirmed that the stringy Euclidean wormhole solutions include instability for flat space in arbitrary dimensions irrespective of the value of integration constant and three-dimensional anti de Sitter space with negative

integration constant where closed form of the wormhole solutions were obtained.

The organization of the paper is as follows. In Sec. II we briefly review the wormhole solutions in d -dimensions. With keeping in mind the analysis of small fluctuations, we perform our calculation with nontrivial lapse function and choose a gauge when we start to solve the equations. In Sec. III we consider small fluctuations of $O(d)$ symmetry and obtained coupled equations of motion for the perturbed fields. Assuming the harmonic perturbation, we find the criterion for the stability and apply this criterion for some special cases. Finally we conclude and discuss our results in Sec. IV.

II. WORMHOLE SOLUTIONS

In this section we recapitulate the instanton and wormhole solutions obtained in Ref. [8]. Bosonic sector of effective supergravity action of type II string theories involves a dilaton ϕ and an axion χ in addition to graviton $g_{\mu\nu}$ while two second-rank antisymmetric tensor fields and self-dual fourth-rank tensor field are turned off. This graviton-dilaton-axion system has been taken into account for various purposes [6, 7] and its generalization to arbitrary dimensions was performed for obtaining instanton and wormhole solutions [8].

Let us begin with the action of the bosonic sector of type II theory. In string frame it is described by

$$S_S = \int d^d x \sqrt{-g_S} \left[e^{-2\phi} (R + 4\nabla_\mu \phi \nabla^\mu \phi) - \frac{1}{2(d-1)!} F_{d-1}^2 - V_S(\phi) \right], \quad (1)$$

where F_{d-1} is an RR $(d-1)$ -form field strength and the subscript S denotes the string frame being used. Note that the dilaton potential is a product of nonperturbative effects so we consider only flat potential (or equivalently cosmological constant) in this paper. Throughout a scale transformation $g_{S\mu\nu} = e^{\frac{4}{d-2}\phi} g_{\mu\nu}$ with $\sqrt{8/(d-2)}\phi \rightarrow \phi$, the action (1) is written in the Einstein frame as

$$S = \int d^d x \sqrt{-g} \left[R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}e^{\sqrt{(d-2)/2}\phi} \frac{1}{(d-1)!} F_{d-1}^2 - V_E(\phi) \right]. \quad (2)$$

Performing a duality transformation from the $(d-2)$ -form antisymmetric tensor field to the axion field χ such as $d\chi = e^{-\sqrt{(d-2)/2}\phi} * F_{d-1}$ in the context of path integral formalism [2], we arrive at the action of our interest in Minkowski signature

$$S_M = \int d^d x \sqrt{-g} \left[R - \frac{1}{2}(\nabla\phi)^2 - \frac{1}{2}e^{b\phi}(\nabla\chi)^2 - V(\phi) \right], \quad (3)$$

where $b = \sqrt{(d-2)/d}$ and thus $b = 2$ in ten-dimensional type IIB theory.

Upon analytic continuation to Euclidean space, the kinetic term of the axion changes sign in the action (3)

$$S_E = \int d^d x \sqrt{g} \left[R - \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}e^{b\phi}(\nabla\chi)^2 - V(\phi) \right]. \quad (4)$$

From here on let us look for the $O(d)$ -symmetric wormhole solution by considering χ as the matter field which supports throat of the wormhole. In general the potential $V(\phi)$ is a function of the dilaton field ϕ , which involves nonperturbative contribution of string theory. Here we take into account only the flat potential independent of ϕ , corresponding to a cosmological constant $\Lambda = V$. Since the dilaton-axion system is invariant under a global $U(1)$ transformation, we have a conserved current $j_\mu = e^{b\phi}\nabla_\mu\chi$;

$$\nabla_\mu j^\mu = 0. \quad (5)$$

Note that this current conservation is paraphrased by the Bianchi identity of $RR(d-1)$ -form field strength in the duality-transformed theory.

First of all, we introduce the most symmetric $O(d)$ -invariant ansatz for the metric

$$ds^2 = n^2(r)dr^2 + a^2(r)d\Omega_{d-1}^2. \quad (6)$$

Subsequently we demand that ϕ and χ depend only on the radial coordinate r . Then the $O(d)$ -symmetric current density $j^0(r)$ is the only nonvanishing component and its conservation (5) lets $\sqrt{g}j^0$ be a constant. In fact, this constant leads to the constant global $U(1)$ charge of the wormhole,

$$\sqrt{g}j^0 = na^{d-1}j^0 = iq, \quad (7)$$

where i results from the rotation of the action to Euclidean signature which coincides with the formal replacement $\chi \rightarrow i\chi$.

From Eq. (7), the axion χ is expressed in terms of the dilaton ϕ and the metric functions n and a :

$$\partial_r\chi = \frac{iqne^{-b\phi}}{a^{d-1}}. \quad (8)$$

With the metric ansatz (6), the nonvanishing components of Ricci tensor are given as

$$R_{rr} = -(d-1)\frac{na'' - n'a'}{na}, \quad (9)$$

$$R_{ij} = -\frac{naa'' - n'aa' + (d-2)na'^2 - (d-2)n^3}{n^3}\delta_{ij}, \quad (10)$$

where the prime (') denotes the differentiation with respect to r . Substituting Eqs. (8)–(10) into the action (4), we obtain the following one-dimensional action after integrating out the angular variables

$$S_E = \text{Vol}(S_{d-1}) \int dr \left[(d-1)(d-2) \frac{a'^2 a^{d-3}}{n} + (d-1)(d-2) n a^{d-3} - \frac{1}{2} \frac{a^{d-1}}{n} \phi'^2 - \frac{1}{2} \frac{q^2 e^{-b\phi} n}{a^{d-1}} - \Lambda n a^{d-1} \right]. \quad (11)$$

From the action (11), we read the equations for ϕ , n and a

$$\phi : \quad \partial_r \left(\frac{a^{d-1}}{n} \partial_r \phi \right) + \frac{b q^2}{2} \frac{n}{a^{d-1}} e^{-b\phi} = 0, \quad (12)$$

$$n : \quad - (d-1)(d-2) \frac{(\partial_r a)^2 a^{d-3}}{n^2} + (d-1)(d-2) a^{d-3} + \frac{1}{2} \frac{a^{d-1}}{n^2} (\partial_r \phi)^2 - \frac{1}{2} \frac{q^2 e^{-b\phi}}{a^{d-1}} - \Lambda a^{d-1} = 0, \quad (13)$$

$$a : \quad \partial_r \left[2(d-2) \frac{(\partial_r a) a^{d-3}}{n} \right] - (d-2)(d-3) \left[\frac{(\partial_r a)^2 a^{d-4}}{n} + n a^{d-4} \right] + \frac{1}{2} \frac{a^{d-2}}{n} (\partial_r \phi)^2 - \frac{1}{2} \frac{q^2 n e^{-b\phi}}{a^d} - \Lambda n a^{d-2} = 0. \quad (14)$$

We set the lapse function to be $n = 1$ as a gauge-fixing condition, which is one of admissible gauges for our purpose [12]. Then the dilaton equation of motion (12) can be integrated by multiplying $a^{d-1} \partial_r \phi$

$$(\partial_r \phi)^2 - \frac{q^2}{a^{2d-2}} e^{-b\phi} - \frac{c}{a^{2d-2}} = 0, \quad (15)$$

where c is an integration constant. Using this integral, the gravitational equations (13) and (14) give

$$1 - (\partial_r a)^2 + \frac{c}{2(d-1)(d-2)a^{2d-4}} - \frac{\Lambda}{(d-1)(d-2)} a^2 = 0. \quad (16)$$

The equation (16) can be solved to find $a(r)$ and then one can find $\phi(r)$ from Eq. (15) and $\chi(r)$ from Eq. (8). The solution will depend on the choice of Λ and c . The cases $\Lambda = 0$, $\Lambda < 0$ and $\Lambda > 0$ correspond to asymptotically flat, anti-de Sitter, and de Sitter space, respectively. The solution for $c \neq 0$ corresponds to a wormhole while the one for $c = 0$ corresponds to an instanton. Since we are interested in the stability of the wormhole solution, we consider the $c \neq 0$ case. Obviously the same equations in Eqs. (15) and (16) can be obtained under the gauge-fixed metric [8]:

$$ds^2 = dr^2 + a^2(r) d\Omega_{d-1}^2, \quad (17)$$

however until now we keep the gauge degree $n(r)$ for the analysis of small fluctuations.

The wormhole solution is obtained when the metric function $a(r)$ has a minimum value a_0 at the neck of the wormhole with $\partial_r a(r) = 0$. From Eq. (16), we have

$$\pm \int_{a_0}^a \frac{da}{\sqrt{1 + \frac{c}{2(d-1)(d-2)a^{2d-4}} - \frac{\Lambda}{(d-1)(d-2)}a^2}} = r, \quad (18)$$

where we choose the position of the neck as the origin, $a(0) = a_0$, without loss of generality. This Euclidean configuration can be continued to Minkowski space. The dilaton can be solved from Eq. (15)

$$\int \frac{d\phi}{\sqrt{q^2 e^{-b\phi} + c}} = \pm \int \frac{dr}{a^{d-1}}. \quad (19)$$

Finally, from Eq. (8), the axion configuration is given by

$$i(\chi(r) - \chi_0) = -q \int dr \frac{e^{-b\phi}}{a^{d-1}}. \quad (20)$$

The solutions for possible values of Λ and c for arbitrary dilaton coupling b were found recently in Ref. [8]. The characters of background solution necessary for our stability analysis can be summarized as follows. For $\Lambda = 0$, one has a minimal size sphere when $c < 0$. The neck of the wormhole and the dilaton profile are given by

$$a_0 = \left[\frac{2(d-1)(d-2)}{|c|} \right]^{-\frac{1}{2d-4}}, \quad (21)$$

$$e^{\frac{b}{2}\phi_0} = \frac{|q|}{|c|^{\frac{1}{2}}} \sin \left[\sin^{-1} \left(\sqrt{\frac{|c|}{q^2}} e^{\frac{b}{2}\phi_\infty} \right) + \frac{\pi}{2} |b| \sqrt{\frac{d-1}{2(d-2)}} \right], \quad (22)$$

where $\phi_\infty = \phi(r = \infty)$. For $\Lambda < 0$, wormhole solution can exist for $c < 0$. The solution can be expressed in terms of elementary functions for the case $d = 3$

$$a_0^2 = \frac{\sqrt{1 + \frac{|\Lambda c|}{2}} - 1}{|\Lambda|}, \quad (23)$$

$$e^{\frac{b}{2}\phi_0} = \frac{|q|}{|c|^{\frac{1}{2}}} \sin \left[\sin^{-1} \left(\sqrt{\frac{|c|}{q^2}} e^{\frac{b}{2}\phi_\infty} \right) \mp \frac{|b|}{2} \left(\frac{\pi}{2} + \sin^{-1} \left(\frac{1}{\sqrt{1 + \frac{|c|\Lambda}{2}}} \right) \right) \right]. \quad (24)$$

The solutions for $d = 4, 5$ can be solved in terms of elliptic integrals. When $c > 5$, no solution in closed form is reported by analytic method, yet. For $\Lambda > 0$, there is no non-singular solution. So we will not consider this case anymore.

III. SMALL FLUCTUATIONS AND STABILITY ANALYSIS

We consider small fluctuations with $O(d)$ symmetry about the obtained nonsingular wormhole solution. Since the important issue in Euclidean wormhole physics is to decide whether the Euclidean wormhole configurations can have purely real contributions to the functional integral or include imaginary contributions, simple study of s -wave perturbation can precede complicated systematic analysis of small fluctuations with angle dependence in this semiclassical approach [9]. To be specific, let us consider

$$n(r) = 1 + \tilde{n}(r), \quad (25)$$

$$a(r) = a_0 + \tilde{a}(r), \quad (26)$$

$$\phi(r) = \phi_0 + \tilde{\phi}(r), \quad (27)$$

where a_0 and ϕ_0 are given in the wormhole solutions obtained from Eqs. (15) and (16). Substitute these into Eq. (11) and take only the bilinear terms in $(\tilde{n}, \tilde{a}, \tilde{\phi})$ of the action. From these terms one can derive the linearized equations for the small fluctuations. In order to keep the consistency with the gauge-fixing condition $n = 1$ we forced, here we also take $\tilde{n} = 0$. The bilinear action is calculated as

$$S_{\text{bil}} = \text{Vol}(S_{d-1}) \int dr a_0^{d-1} \left[A_0 \tilde{a}'^2 + B_0 \tilde{\phi}'^2 + C_0 \tilde{a} \tilde{\phi}' + D_0 \tilde{a} \tilde{\phi} + E_0 \tilde{a}^2 + F_0 \tilde{\phi}^2 \right], \quad (28)$$

with boundary terms which are not relevant for our analysis. Here A_0, \dots, F_0 are given as

$$\begin{aligned} A_0 &= \frac{(d-1)(d-2)}{a_0^2}, \\ B_0 &= -\frac{1}{2}, \\ C_0 &= -\frac{(d-1)}{a_0} \phi_0', \\ D_0 &= -\frac{1}{2} \frac{(d-1)}{a_0} b Q^2, \\ E_0 &= \frac{(d-1)(d-2)(d-3)(d-4)}{2} \frac{1}{a_0^4} - \frac{d(d-1)}{4a_0^2} Q^2 - \frac{(d-1)(d-2)}{2} \left(\frac{\phi_0'^2}{2a_0^2} + \frac{\Lambda}{a_0^2} \right), \\ F_0 &= -\frac{b^2}{4} Q^2, \end{aligned} \quad (29)$$

where

$$Q^2 = \frac{q^2 e^{-b\phi_0}}{a_0^{2(d-1)}}. \quad (30)$$

The equations of motion for \tilde{a} and $\tilde{\phi}$ are summarized by

$$-\tilde{a}'' + \frac{C_0}{2A_0}\tilde{\phi}' + \frac{E_0}{A_0}\tilde{a} + \frac{D_0}{2A_0}\tilde{\phi} = 0, \quad (31)$$

$$-\tilde{\phi}'' + C_0\tilde{a}' - D_0\tilde{a} - 2F_0\tilde{\phi} = 0. \quad (32)$$

Since they are coupled linear differential equations of second-order, we employ the assumption of harmonic form which looks appropriate for testing existence of negative modes

$$\tilde{a} = \tilde{a}_0 e^{i\omega r}, \quad \tilde{\phi} = \tilde{\phi}_0 e^{i\omega r}.$$

Then the normal modes of the coupled equations are determined by a 2×2 matrix equation

$$\begin{pmatrix} \omega^2 + \frac{E_0}{A_0} & \frac{1}{2A_0}(D_0 + i\omega C_0) \\ -(D_0 - i\omega C_0) & \omega^2 - 2F_0 \end{pmatrix} \begin{pmatrix} \tilde{a}_0 \\ \tilde{\phi}_0 \end{pmatrix} = 0. \quad (33)$$

From the determinant of the matrix equation we obtain

$$\omega^4 + \alpha\omega^2 + \beta = 0, \quad (34)$$

where

$$\alpha = \frac{E_0}{A_0} - 2F_0 + \frac{C_0^2}{2A_0}, \quad \beta = \frac{D_0^2}{2A_0} - \frac{2E_0F_0}{2A_0}. \quad (35)$$

The explicit forms of α and β are calculated as, upon eliminating ϕ'_0 using Eq. (15),

$$\alpha = \frac{(d-3)(d-4)}{2a_0^2} + \frac{1}{4} \frac{d}{d-2} \frac{c}{a_0^{2(d-1)}} + \frac{b^2}{2} Q^2 - \frac{\Lambda}{2}, \quad (36)$$

$$\beta = \frac{b^2}{2} Q^2 \left(\frac{(d-3)(d-4)}{2a_0^2} - \frac{1}{4} \frac{c}{a_0^{2(d-1)}} - \frac{1}{4} \frac{d-1}{d-2} Q^2 - \frac{\Lambda}{2} \right). \quad (37)$$

The condition for the wormhole solution not to have a negative mode over the perturbation is that Eq. (34) has all real roots. The existence of imaginary part in ω means that $e^{i\omega r}$ can grow exponentially. This tells instability of the solution. The condition for ω to have all real roots is equivalent to ω^2 to have all non-negative real roots. So the criterion for the stability can be written as

$$\alpha < 0, \quad \beta > 0, \quad \alpha^2 - 4\beta > 0. \quad (38)$$

A. Flat space

In this case, it has been pointed out that the wormhole solution exists for $c < 0$ [8]. Taking $\Lambda = 0$ in Eqs. (36) and (37), the conditions in Eq. (38) become

$$\frac{-3(d-3)}{a_0^2} + \frac{b^2}{2}Q^2 < 0, \quad (39)$$

$$\frac{b^2Q^2}{2} \left(\frac{d^2 - 5d + 7}{a_0^2} - \frac{1}{4} \frac{d-1}{d-2} Q^2 \right) > 0, \quad (40)$$

$$\frac{b^2}{2} \left(\frac{b^2}{2} + \frac{d-1}{d-2} \right) Q^4 - b^2(2d^2 - 7d + 8) \frac{Q^2}{a_0^2} + \frac{9(d-2)^2}{a_0^4} > 0. \quad (41)$$

We restrict ourselves to the type II case where b is given by $b = \sqrt{(d-2)/2}$ for simplicity. By solving the above inequalities in terms of Q , we obtain a constraint on the parameters for stability condition

$$d = 3 : \quad Q^2 < \frac{8}{a_0^2}, \quad (42)$$

$$d = 4 : \quad Q^2 < \frac{2}{a_0^2}, \quad (43)$$

$$d \geq 5 : \quad Q^2 < \frac{4(d-2)}{d^2 a_0^2} (2d^2 - 7d + 8 - \sqrt{(2d^2 - 7d + 8)^2 - 9d^2}). \quad (44)$$

Using Eqs. (21), (22) and (30), the stability condition is expressed in terms of U(1) charge q and the parameter of wormhole solution c

$$d = 3 : \quad \sin \left(\sin^{-1} \left(\sqrt{\frac{|c|}{q^2}} e^{\frac{b}{2}\phi_\infty} \right) + \frac{\sqrt{2}\pi}{4} \right) > \sqrt{2}, \quad (45)$$

$$d = 4 : \quad \sin \left(\sin^{-1} \left(\sqrt{\frac{|c|}{q^2}} e^{\frac{b}{2}\phi_\infty} \right) + \frac{\sqrt{3}\pi}{4} \right) > \sqrt{\frac{3}{2}}, \quad (46)$$

$$\begin{aligned} d \geq 5 : \quad & \sin \left(\sin^{-1} \left(\sqrt{\frac{|c|}{q^2}} e^{\frac{b}{2}\phi_\infty} \right) + \frac{\sqrt{d-1}\pi}{4} \right) \\ & > \left[\frac{d-1}{18} \left(2d^2 - 7d + 8 + \sqrt{(2d^2 - 7d + 8)^2 - 9d^2} \right) \right]^{\frac{1}{2}}. \end{aligned} \quad (47)$$

The right-hand side of Eq. (47) is larger than one for $d \geq 5$. Therefore Eqs. (45), (46) and (47) cannot hold for any choice of c and q . Thus we conclude that the wormhole solution with $\Lambda = 0$ shows unstable behavior under the small fluctuations of $O(d)$ symmetry.

B. Anti de Sitter space

For $\Lambda < 0$, wormhole solution can exist for $c < 0$. We consider the $d = 3$ case where the wormhole solution can be expressed in terms of elementary functions. Here we also consider the type II case where b is given by $b = \sqrt{(d-2)/2}$. Taking $d = 3$ and $b = \sqrt{1/2}$ in Eqs. (36) and (37), we have

$$\alpha = \frac{3}{4} \frac{c}{a_0^4} + \frac{Q^2}{4} - \frac{\Lambda}{2}, \quad (48)$$

$$\beta = \frac{1}{4} Q^2 \left(-\frac{1}{4} \frac{c}{a_0^4} - \frac{1}{2} Q^2 - \frac{\Lambda}{2} \right). \quad (49)$$

From Eq. (16), we have for $d = 3$

$$1 + \frac{c}{4a_0^2} - \frac{\Lambda}{2} a_0^2 = 0. \quad (50)$$

Using Eq. (50), α and β can be simplified further

$$\alpha = -\frac{3}{a_0^2} + \frac{Q^2}{4} + \Lambda, \quad (51)$$

$$\beta = \frac{1}{4} Q^2 \left(\frac{1}{a_0^2} - \frac{1}{2} Q^2 - \Lambda \right). \quad (52)$$

With the above choice of α and β , we have

$$\alpha^2 - 4\beta = \left(\frac{3}{4} Q^2 - \frac{3}{a_0^2} - \Lambda \right)^2 + \frac{2}{a_0^2} > 0. \quad (53)$$

The condition $\alpha^2 - 4\beta > 0$ is satisfied automatically. Thus the constraint for the stability is obtained, from $\alpha < 0$ and $\beta > 0$, as

$$Q^2 < 2 \left(\frac{1}{a_0^2} + |\Lambda| \right). \quad (54)$$

Using Eqs. (23), (24) and (30), this condition can be written as

$$\sin \left[\sin^{-1} \left(\sqrt{\frac{|c|}{q^2}} e^{\frac{b}{2}\phi_\infty} \right) \mp \frac{1}{2\sqrt{2}} \left(\frac{\pi}{2} + \sin^{-1} \left(\frac{1}{\sqrt{1 + \frac{|c|\Lambda}{2}}} \right) \right) \right] > \left(1 + \frac{1}{\sqrt{1 + \frac{|c|\Lambda}{2}}} \right)^{\frac{1}{2}}. \quad (55)$$

The right-hand side of Eq. (55) is always larger than one, so one can conclude that the wormhole solution is also unstable for three-dimensional anti de Sitter space.

IV. DISCUSSION

In this paper we have studied the classical stability of stringy wormhole solutions. For the small fluctuations with $O(d)$ symmetry, the analysis to linear order results in two coupled differential equations of second order. Under the assumption of harmonic form perturbation, we obtained a condition for the criterion of stability of the Euclidean wormhole solutions, which is a function of the dimensionality of the spacetime, the dilaton coupling b , the cosmological constant Λ as well as the integration constant c distinguishing instanton and wormhole. As a concrete test we applied our criterion to type II case where the coupling is given by $b = \sqrt{(d-2)/2}$ and showed that every wormhole solution expressed in terms of elementary functions is unstable. It would be also interesting if one could find any example in string theory where b is in the range where the wormhole solution is stable under the classical perturbation.

Acknowledgments

We would like to thank S.P. Kim for useful discussion. J.Y.K. would like to thank the Department of Physics at University of the Pacific for hospitality during his visit. This work was supported by the Korea Research Foundation Grant (KRF-2001-015-DP0082).

-
- [1] S. B. Giddings and A. Strominger, Nucl. Phys. B **306**, 890 (1988).
 - [2] K. Lee, Phys. Rev. Lett. **61**, 263 (1988); B. Grinstein, Nucl. Phys. B **321**, 439 (1989); L. F. Abbott and M. B. Wise, *ibid.* B **325**, 687 (1989); S. Coleman and K. Lee *ibid.* B **329**, 387 (1990).
 - [3] A. Hosoya and W. Ogura, Phys. Lett. B **225**, 117 (1989).
 - [4] R. C. Myers, Phys. Rev. D **38**, 1327 (1988); K. Yosida, S. Hienzaki and K. Shiraishi, Phys. Rev. D **42**, 1973 (1990).
 - [5] H. Fukutaka, K. Ghoroku and K. Tanaka Phys. Lett. B **222**, 191 (1989).
 - [6] S. B. Giddings and A. Strominger, Phys. Lett. B **230**, 46 (1989).
 - [7] S. Rey, Phys. Rev. D **43**, 526 (1991).
 - [8] M. Gutperle and W. Sabra, Nucl. Phys. B **647**, 344 (2002) hep-th/0206153.

- [9] V. A. Rubakov and O. Shvedov, Phys. Lett. B **383**, 258 (1996); gr-qc/9608065.
- [10] J. Y. Kim, H. W. Lee and Y. S. Myung, Phys. Rev. D **56**, 6684 (1997).
- [11] S. Chandrasekhar, *The Mathematical Theory of Black Hole* (Oxford University Press, New York, 1983); O. J. Kwon, Y. D. Kim, Y. S. Myung, B. H. Cho, and Y. J. Park, Phys. Rev. D **34** 333 (1986); Int. J. Mod. Phys. A **1**, 709 (1986).
- [12] T. Tanaka and M. Sasaki, Prog. Theor. Phys. **88**, 503 (1992).